SPECIALIST MATHEMATICS

(Year 11 and 12)

UNIT A

A1: Combinatorics

Permutations:

- problems involving permutations
- use the multiplication principle and factorial notation
- permutations and restrictions with or without repeated objects

The inclusion-exclusion principle for the union of two sets and three sets:

Combinations

- problems involving combinations
- $\binom{n}{r}$ or ${}^{n}C_{r}$
- Pascal's triangle

A2: Vectors in the plane

Representing vectors:

- magnitude and direction of a vector
- vectors - displacement and velocity
- scalar multiple of a vector
- triangle rule to find the sum and difference of two vectors.

Algebra of vectors:

- ordered pair notation and column vector notation to represent a vector
- unit vectors and the perpendicular unit vectors
- vector in component form
- addition and subtraction of vectors in component form
- multiplication by a scalar of a vector in component form
- scalar (dot) product
- parallel and perpendicular vectors
- projections of vectors
- problems involving displacement, force and velocity

A3: Geometry

The nature of proof:

- implication, converse, equivalence, negation, contrapositive
- proof by contradiction

- use the symbols for implication (⇒), equivalence (⇔), and equality (=)
- use the quantifiers ‘for all’ and ‘there exists’
- examples and counter-examples

Circle properties and their proofs:

- Angles in a semicircle / circle
- Chords
- Alternate segment theorem
- Secant and tangent to a circle
- Problems finding unknown angles and lengths

Geometric proofs using vectors in the plane:

- Diagonals of a parallelogram
- Midpoints of a quadrilateral

UNIT B

B1: Trigonometry

The basic trigonometric functions:

- solutions of sin, cos or tan
- graph functions of sin, cos, or tan
- compound angles
- angle sum, difference and double angle identities

The reciprocal trigonometric functions, secant, cosecant and cotangent and their graphs:

- graph and simple transformations

Trigonometric identities:

- Pythagorean identities
- Products of sines and cosines expressed as sums and differences
- solve trigonometric equations
- trigonometric identities

Applications of trigonometric functions:

- model periodic motion using sine and cosine functions
- relevance of the period and amplitude of these functions

B2: Matrices

Matrix arithmetic:

- matrix definition and notation
- addition and subtraction of matrices, scalar multiplication, matrix multiplication, multiplicative identity and inverse
- determinant and inverse of 2 × 2 matrices and solve matrix equations of the form AX = B

Transformations in the plane:

- translations and representation as column vectors
- basic linear transformations: dilations, rotations and reflection
- representations of the transformations by 2 × 2 matrices
- transformations to points in the plane and geometric objects
- composition of linear transformations and the corresponding matrix products
- inverses of linear transformations and the relationship with the matrix inverse
- relationship between the determinant and the effect of a linear transformation on area
- geometric results by matrix multiplications

Topic 3: Real and complex numbers

Proofs involving numbers:

- Rational and irrational numbers:
- rational numbers as terminating or recurring decimals and vice versa
- prove irrationality by contradiction

Proof by mathematical induction:

- nature of inductive proof including the 'initial statement' and inductive step
- problems related to sums and divisibility

Complex numbers:

- imaginary number
- complex numbers in the form a + bi
- complex conjugates
- complex-number arithmetic: addition, subtraction, multiplication and division.

The complex plane:

- complex numbers as points in a plane, Cartesian coordinates
- addition of complex numbers as vector addition in the complex plane
- location of complex conjugates in the complex plane

Roots of equations:

- real quadratic equations
- complex conjugate solutions of real quadratic equations
- linear factors of real quadratic polynomials.

UNIT C

C1: Complex numbers

Cartesian forms:

- review real and imaginary parts $Re(z)$ $Re(z)$and $Im(z)$ of a complex number z
- review Cartesian form
- review complex arithmetic using Cartesian forms

Complex arithmetic using polar form:

- prove basic identities involving modulus and argument
- Cartesian and polar form conversion
- multiplication, division, and powers of complex numbers in polar form and the geometric interpretation
- De Moivre's theorem for integral powers

The complex plane (the Argand plane):

- addition of complex numbers as vector addition in the complex plane
- multiplication as a linear transformation in the complex plane
- identify subsets of the complex plane

Roots of complex numbers

- n^{th} roots of unity and their location on the unit circle
- n^{th}roots of complex numbers and their location in the complex plane

Factorisation of polynomials:

- factor theorem and the remainder theorem for polynomials
- conjugate roots for polynomials with real coefficients
- solve simple polynomial equations.

C2: Functions and sketching graphs

Functions:

- composition of two functions
- one-to-one function
- consider inverses of one-to-one function
- reflection property of the graph and its inverse

Sketching graphs:

- absolute value for the real number and graph
- relationships between the graph
- sketch the graphs of simple rational functions

C3: Vectors in three dimensions

The algebra of vectors in three dimensions:

- concepts of three dimensional vectors
- geometric results in a plane and simple proofs in three-dimensions

Vector and Cartesian equations:

- Cartesian coordinates for three-dimensional space, including plotting points and the equations of spheres
- vector equations of curves in 2D and 3Dinvolving a parameter, and determine a ‘corresponding’ Cartesian equation in the 2D
- vector equation of a straight line and straight-line segment, given the position of two points, or equivalent information, in both two and three dimensions
- position of two particles each described as a vector function of time, and determine if their paths cross or if the particles meet
- cross product to determine a vector normal to a given plane
- vector and Cartesian equations of a plane and of regions in a plane

Systems of linear equations:

- general form of a system of linear equations in several variables,
- elementary techniques of elimination to solve a system of linear equations
- three cases for solutions of systems of equations –
    - a unique solution
    - no solution, and
    - infinitely many solutions
- geometric interpretation of a solution of a system of equations with three variables

Vector calculus:

- consider position of vectors as a function of time
- Cartesian equation of a path given as a vector equation in two dimensions including ellipses and hyperbolas
- differentiate and integrate a vector function with respect to time
- equations of motion of a particle travelling in a straight line with both constant and variable acceleration
- vector calculus -- motion in a plane including projectile and circular motion

UNIT D

D1: Integration and applications of integration

Integration techniques:

- integrate using the trigonometric identities
- integrate expressions of the form $f\big(g(x)\big)g'(x)$
- integrate $\int \frac{1}{x}dx = \ln|x| + c$, for $x \neq 0 x \neq 0$
- inverse trigonometric functions: arcsine, arccosine and arctangent
- derivative of the inverse trigonometric functions: arcsine, arccosine and arctangent
- integrate expressions of the form $\frac{\pm 1}{\sqrt{a^2-x^2}}$ and $\frac{a}{a^2+x^2}$
- partial fractions where necessary for integration in simple cases
- integrate by parts

Applications of integral calculus:

- areas between curves determined by functions
- determine volumes of solids of revolution about either axis
- numerical integration using technology
- probability density function, $f(t) = \lambda e^{-\lambda t}$ for $t \geq 0$, of the exponential random variable with parameter $\lambda > 0$, and
- exponential random variables and associated probabilities and quantiles to model data and solve practical problems

Topic 2: Rates of change and differential equations

- implicit differentiation to determine the gradient of curves
- Related rates as instances of the chain rule
- solve simple first-order differential equations
- examine slope (direction or gradient) fields of a first order differential equation
- formulate differential equations including the logistic equation e.g. chemistry, biology and economics, in situations where rates are involved

Modelling motion:

- momentum, force, resultant force, action and reaction
- constant and non-constant force
- understand motion of a body under concurrent forces
- solve problems involving motion in a straight line with both constant and non-constant acceleration, including simple harmonic motion and acceleration

Topic 3: Statistical inference

Sample means:

- examine the concept of the sample mean $\bar{X}$ as a random variable whose value varies between samples where *X* is a random variable with mean μ and the standard deviation σ
- random sampling, distribution of $\bar{X}$ across samples of a fixed size n, including its mean μ, its standard deviation $\sigma/\sqrt{n}$ (where μ and σ are the mean and standard deviation of X), and its approximate normality if n is large
- approximate standard normality of large samples ($n \geq 30$), where s is the sample standard deviation.

Confidence intervals for means:

- concept of an interval estimate for a parameter associated with a random variable
- confidence interval, as an interval estimate for μ ,the population mean, where z is the appropriate quantile for the standard normal distribution
- illustrate variations in confidence intervals between samples
- use $\bar{x}$ and s to estimate μ and σ,
- collect data and construct an approximate confidence interval
- estimate a mean and report on survey procedures and data quality

References:

1. For full details about any ACT curriculum, please refer to ACT Board of Senior Secondary Studies (BSSS Courses); http://www.bsss.act.edu.au/curriculum/courses